Comment on "Resonant Spectra and the Time Evolution of the Survival and Nonescape Probabilities"

Some time ago, García-Calderón, Mateos and Moshinsky [1] investigated the time evolution of the quantum decay of a state initially located within an interaction region of finite range. In particular, they showed that the survival $S(t)$ and nonescape $P(t)$ probabilities behave differently at large times: $S(t) \sim t^{-3}$, whereas $P(t) \sim t^{-1}$ when $t \to \infty$. The purpose of this Comment is to show, following their analysis, that $P(t) \sim t^{-3}$ also — a result already reported in the literature [2].

According to Ref. [1], $P(t) \sim t^{-1}$ asymptotically because (Eq. (21); equation numbering follows Ref. [1])

$$\Delta \equiv \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} \frac{C_r^* C_s I_{rs}}{k_r^* k_s} \neq 0.$$

However, $\Delta = 0$, instead. To prove this, we use the definition of I_{rs} [Eq. (15)],

$$I_{rs} = \int_0^R u_r^*(r) u_s(r)\, dr,$$

to rewrite Δ as

$$\int_0^R f^*(r) f(r)\, dr, \qquad f(r) \equiv \sum_{n=-\infty}^{\infty} \frac{C_n u_n(r)}{k_n}.$$

Using the definition of C_n [Eq. (12)], the function $f(r)$ can be rewritten as

$$f(r) = \int_0^R \psi(r',0) \left(\sum_{n=-\infty}^{\infty} \frac{u_n(r') u_n(r)}{k_n} \right) dr',$$

where $\psi(r,0)$ is the wave function at $t = 0$. Since the term in parentheses vanishes [Eq. (7)], it follows that $f(r) = 0$, which completes the proof.

To prove that $P(t) \sim t^{-3}$, it is convenient to work directly with the time-dependent Green function, $g(r,r';t)$, which, for $(r,r') < R$, can be written as [Eq. (9)]

$$g(r,r';t) = \sum_{n=-\infty}^{\infty} u_n(r) u_n(r') M(k_n,t).$$

Using Eqs. (18) and (19), and the relations $k_{-n} = -k_n^*$, $u_{-n}(r) = u_n^*(r)$, one can find the asymptotic expansion of $g(r,r';t)$:

$$\begin{aligned} g(r,r';t) \approx & \sum_{n=1}^{\infty} u_n(r) u_n(r')\, e^{-ik_n^2 t} \\ & + \frac{A}{t^{1/2}} \sum_{n=-\infty}^{\infty} \frac{u_n(r) u_n(r')}{k_n} \\ & + \frac{B}{t^{3/2}} \sum_{n=-\infty}^{\infty} \frac{u_n(r) u_n(r')}{k_n^3} + \ldots, \end{aligned}$$

where A and B are constants. Again, because of Eq. (7), the coefficient of $At^{-1/2}$ cancels out exactly and, therefore, $g(r,r';t) \sim t^{-3/2}$ when $t \to \infty$. Since

$$P(t) = \int_0^R \psi^*(r,t) \psi(r,t)\, dr,$$

and

$$\psi(r,t) = \int_0^R g(r,r';t) \psi(r',0)\, dr',$$

it follows that $P(t) \sim t^{-3}$ asymptotically.

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[1] G. García-Calderón, J. L. Mateos, and M. Moshinsky, Phys. Rev. Lett. **74**, 337 (1995).
[2] See, for example, H. M. Nussenzveig, *Causality and Dispersion Relations* (Academic Press, New York, 1972), p. 183.